AUTONOMOUS PERPENDICULAR AND PARALLEL PARKING USING MULTI-SENSOR BASED CONTROL: CONVERGENCE ANALYSIS

David Pérez-Morales, Olivier Kermorgant, Salvador Domínguez-Quijada and Philippe Martinet

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Modeling and Notation
Car-Like Robot Rear-Wheel Driving

Figure: Kinematic model diagram for a car-like rear-wheel driving robot

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta \\
\sin \theta \\
\tan \phi/l_{wb} \\
0
\end{bmatrix} v +
\begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix} \dot{\phi}
\]

(1)

Where \( v \) and \( \dot{\phi} \) are the driving and steering velocities.
Experimental Setup

Velocity, direction of travel, steering and turning signals can be controlled by computer.

Figure: Robotized Renault ZOE
Multi-sensor modeling

In a static environment, the sensor feature derivative can be expressed as:

\[ \dot{s}_i = L_i v_i = L_i W_m v_m \]

where

\[ L_s = LW_m = \begin{bmatrix} L_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & L_k \end{bmatrix} \begin{bmatrix} W_m^1 \\ \vdots \\ W_m^k \end{bmatrix} \]

and

\[ \dot{s} = L_s v_m \]

Under a planar world assumption:

\[ \dot{s}_i = L_{ir} v_{ir} = L_{ir} W_{mr} v_{mr} \]

where

\[ v_{mr} = [v_{x_m}, v_{y_m}, \dot{\theta}]^T \]

Kermorgant and Chaumette, “Dealing with constraints in sensor-based robot control”
Assuming $v_{ym} = 0$ (no slipping nor skidding)

\[
v = [v_{xm}, \dot{\theta}]^T \\
\text{dim}(L_v) = (d \times 2)
\]

where $v_{xm} = v$ and $L_v$ is the corresponding sub-matrix extracted from $L_{sr}$.

Figure: Kinematic model diagram for a car-like rear-wheel driving robot
The weighted multi-sensor error signal is defined as:

\[ e_H = He \]  

(7)

where \( e = s - s^* \) is the difference between the current sensor signal \( s \) and its desired value \( s^* \) and \( H \) is a diagonal positive semi-definite weighting matrix that depends on the current value of \( s \). Its associated interaction matrix is \( L_H = HL_s \).
Perception
Perception

Sensor fusion → Filtering and downsampling → Segmentation and clustering → Orientation extraction → Bounding box extraction
Extraction of empty parking place
Parking spots

Figure: Parking spot model for reverse || parking maneuvers

Figure: Parking spot model for forward || parking maneuvers

Table: Pair of points through which each line passes

<table>
<thead>
<tr>
<th>Line</th>
<th>Perpendicular</th>
<th>Parallel (reverse)</th>
<th>Parallel (forward)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$'L_1$</td>
<td>($'p_5$, $'p_6$)</td>
<td>($'p_5$, $'p_6$)</td>
<td>($'p_5$, $'p_6$)</td>
</tr>
<tr>
<td>$'L_2$</td>
<td>($'p_1$, $'p_4$)</td>
<td>($'p_3$, $'p_4$)</td>
<td>($'p_1$, $'p_2$)</td>
</tr>
<tr>
<td>$'L_3$</td>
<td>($'p_3$, $'p_4$)</td>
<td>($'p_1$, $'p_4$)</td>
<td>($'p_1$, $'p_4$)</td>
</tr>
<tr>
<td>$'L_4$</td>
<td>($'p_1$, $'p_2$)</td>
<td>($'p_1$, $'p_2$)</td>
<td>($'p_3$, $'p_4$)</td>
</tr>
</tbody>
</table>
Interaction Model
The sensor signals $s_{iL_j}$ and reduced interaction matrix $L_{iL_j}$ are defined respectively as:

$$ s_{iL_j} = \begin{bmatrix} \underline{u}_j(1), \underline{u}_j(2), \underline{h}_j(3) \end{bmatrix}^T $$  \hspace{1cm} (8)$$

$$ L_{iL_j} = \begin{bmatrix}
0 & 0 & i\underline{u}_j(2) \\
0 & 0 & -i\underline{u}_j(1) \\
-i\underline{u}_j(2) & i\underline{u}_j(1) & 0 \\
\end{bmatrix} $$  \hspace{1cm} (9)$$

Figure: Sensors’ configuration and sensor features
Task sensor features

\[ \mathbf{s}^t = \left[ \mathbf{s}_{i, L_1}, \mathbf{s}_{i, L_2} \right]^T \]  \hspace{1cm} (10)

\( \mathbf{s}^t \) is obtained from \( S_1 \) for forward maneuvers and from \( S_2 \) for reverse ones. The corresponding interaction matrix is defined as:

\[
\mathbf{L}^t = \frac{\mathbf{L}_L + \mathbf{L}_L^*}{2}
\]  \hspace{1cm} (11)

where \( \mathbf{L}_L = \left[ \mathbf{L}_{i, L_1}, \mathbf{L}_{i, L_2} \right]^T \) and \( \mathbf{L}_L^* \) is equal to the value of \( \mathbf{L}_L \) at the desired pose.
Weighting of the task sensor features

The associated weighting matrix $H_t$ is defined as:

$$H^t = \text{diag}(h_1^t, h_2^t, h_3^t, h_4^t, h_5^t, h_6^t)$$  \hspace{1cm} (12)

where the values $h_3^t$ and $h_6^t$ are constant while the values of $h_i^t \hspace{0.5cm} \forall \hspace{0.2cm} i = 1, 2, 4, 5$ are computed using the following smooth weighting function:
Constraints

Constrained sensor features

\[ s^c = [s_3, \ldots, s_8]^T \] (13)

The corresponding interaction matrix:

\[ L^c = [L_3, \ldots, L_8]^T \] (14)

Table: Constraints features for \( \perp \) maneuvers

<table>
<thead>
<tr>
<th>( s_i )</th>
<th>Reverse</th>
<th>Forward</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_3 )</td>
<td>([^3h_2(3), ^3y_2, ^3d_{lat_2}]^T)</td>
<td>(^3y_3)</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>–</td>
<td>(^4h_2(3))</td>
</tr>
<tr>
<td>( s_5 )</td>
<td>(^5h_3(3))</td>
<td>([^5h_2(3), ^5h_4(3), ^5d_2]^T)</td>
</tr>
<tr>
<td>( s_6 )</td>
<td>([^6h_2(3), ^6h_3(3)]^T)</td>
<td>–</td>
</tr>
</tbody>
</table>

Table: Constraints features for \( \parallel \) maneuvers

<table>
<thead>
<tr>
<th>( s_i )</th>
<th>Reverse</th>
<th>Forward</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_3 )</td>
<td>([^3h_2(3), ^3y_2, ^3d_{lat_2}]^T)</td>
<td>([^3y_3, ^3d_{lat_3}]^T)</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>([^4y_2, ^4d_2]^T)</td>
<td>(^4h_2(3))</td>
</tr>
<tr>
<td>( s_5 )</td>
<td>–</td>
<td>(^5h_2(3))</td>
</tr>
<tr>
<td>( s_6 )</td>
<td>(^6h_2(3))</td>
<td>–</td>
</tr>
<tr>
<td>( s_7 )</td>
<td>(^7h_3(3))</td>
<td>(^7h_3(3))</td>
</tr>
<tr>
<td>( s_8 )</td>
<td>(^8h_3(3))</td>
<td>(^7h_3(3))</td>
</tr>
</tbody>
</table>

Figure: Radial constraints: all the radii define concentric arcs with center at ICR
Constraints (reverse perpendicular case)

Figure: Constraints required for reverse $\perp$ parking maneuvers

Constrained sensor features

\[ s^c = [s_3, s_5, s_6]^T \]  \hspace{1cm} (15)

The corresponding interaction matrix:

\[ L^c = [L_3, L_5, L_6]^T \]  \hspace{1cm} (16)
Control law

\[ \mathbf{v} = \text{argmin} ||\mathbf{L}_H^t \mathbf{v} + \lambda \mathbf{e}_H^t||^2 \]
\[ \text{s.t. } \mathbf{A} \mathbf{v} \leq \mathbf{b} \]  

with:

\[ \mathbf{A} = [\mathbf{L}^c, -\mathbf{L}^c]^T \]  
\[ \mathbf{b} = [\alpha (\mathbf{s}^{c^+} - \mathbf{s}^c), -\alpha (\mathbf{s}^{c^-} - \mathbf{s}^c)]^T \]

where \( \alpha \) is a gain constant, \( \lambda \) is the control gain and \([\mathbf{s}^{c^-}, \mathbf{s}^{c^+}]\) is the desired interval in which we want to keep \( \mathbf{s}^c \).
Bounding the control signals

The control signals $v$ and $\phi$ and their increments are bounded as shown below:

1. $|v| \leq v_{max}$  \hspace{2cm} (20)
2. $|\phi| \leq \phi_{max}$  \hspace{2cm} (21)
3. $(v_{n-1} - \Delta_{dec}) \leq v_n \leq (v_{n-1} + \Delta_{acc})$ \hspace{2cm} (22)
4. $(\phi_{n-1} - \Delta_{\phi}) \leq \phi_n \leq (\phi_{n-1} + \Delta_{\phi})$ \hspace{2cm} (23)

Figure: Distance to stop line

Figure: Deceleration profile
Results
Convergence Analysis - Exhaustive Simulations

Convergence Analysis - Reverse Perpendicular Case ($\theta_{T=0} = 0^\circ$)

Figure: Reverse $\perp$ case, spot length = 4m and width = 2.7m
Convergence Analysis - Exhaustive Simulations

Convergence Analysis - Forward Perpendicular Case ($\theta_{T=0} = 0^\circ$)

Figure: Forward $\perp$ case, spot length = 4m and width = 2.7m
Convergence Analysis - Reverse Parallel Case ($\theta_{T=0} = 0^\circ$)

Figure: Reverse $\parallel$ case, spot length = 7.5m and width = 2.3m
Convergence Analysis - Exhaustive Simulations

Convergence Analysis - Forward Parallel Case ($\theta_{T=0} = 0^\circ$)

Figure: Forward $\parallel$ case, spot length = 11.5m and width=2.3m
Real Experimentation

Figure: Reverse ⊥ parking maneuver (https://youtu.be/Lm5-pFiV5pA)
Convergence Analysis - Real Experimentation

The initial position of the vehicle (denoted by a black \( \times \)) lies inside of the region of attraction (ROA).

Figure: Reverse \( \perp \) case, spot length = 4.2m and width = 2.8m
Real Experimentation - Parking Maneuver Signals

(a) Control signals

(b) Task error signal

(c) Task features’ weights

(d) Constraints sensor signals

Figure: Reverse ⊥ parking maneuver signals
Conclusions and Future Work
Conclusions and Future Work

Conclusions:

- The presented technique has been proven to be very versatile and robust.
- The regions of attraction (ROAs) are quite extensive and their boundaries seem natural.

Future work:

- Validate the approach for other parking scenarios by real experimentation.
- To be able to park with multiple maneuvers.